Lebanese American University **Spring 2006**

Byblos

Probability and Statistics **Date:** 12/05/2006
Test #2 **Duration:** 1h 30

Nota Bene. Use 5 digits in the fractional part of your answers.

1. A couple decides they will continue to have children until they have three males. Assuming that $P(\text{male}) = 0.48$. What is the probability that their third male is their fifth child.

2. It is estimated that 80% of the 12000 voting residents of a town are against a new sales tax. If 20 elligible voters are selected at random and asked their opinion, what is the probability that at most 5 favor to new tax?

3. A coin is biased so that $P(\text{head}) = 0.7$. We consider the following two (consecutive) statistical experiments:

   - **First experiment:** We flip the coin 15 times.
   - **Second experiment:** We flip the coin n times, where n is the number of heads obtained in the first experiment.

   (a) Find the probability that we obtain at least 12 heads in the first experiment.

   (b) Given that we obtain 10 heads in the first experiment, what is the probability that the number of obtained heads in the second experiment is less than 8.

   (c) Find the probability that we obtain 15 heads in the first experiment and 13 heads in the second experiment.

   (d) Find the probability that we obtain 14 heads in the second experiment.

   (e) *Bonus question.* We consider Y the number of obtained heads in the second experiment. Find the probability distribution of Y.

4. Let X be a continuous random variable such that its density function is

$$f(x) = \begin{cases} ke^{-x}, & x \geq 2 \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) Evaluate k.

   (b) Find μ_X and σ_X^2.

   (c) Compute $P(\mu_X - 2\sigma_X < X < \mu_X + 2\sigma_X)$ and compare with the result given by Chebyshev's theorem.

$\longrightarrow$

(d) Now we consider the random variable Y defined by $Y = [X]$, where $[\cdot]$ is the integer floor function. We recall that for $x \in \mathbb{R}$, $[x]$ is the *greatest integer less than or equal* to x.

i. Find the probability distribution of Y.

ii. Find μ_Y.
(*Hint*: Use that $\sum_{n=0}^{\infty} nx^n = \frac{x}{(1-x)^2}$ for $x \in]-1, 1[$)

MARKS : 1. [15] **2.** [15] **3.** [30] **4.** [40] Bonus question [10]

Lebanese American University **Spring 2007**

Probability and Statistics
Test #2

Date: 17/05/2007
Duration: 1h 30

Nota Bene.

- *Use 5 digits in the fractional part of your answers.*
- *In all the probability questions of exercises 1 and 4, you must use a discrete probability distribution by giving its name.*

1. Computer technology has produced an environment in which robots operate with the use of microprocessors. The probability that a robot fails during any 6 hours shift is 0.2. What is the probability that a robot will operate at most 4 shifts before it fails?

2. We consider the following gambling game. First the player must pay 15 \$ to play the game. The player begins the game by flipping a coin two times:
   - If he obtains two heads or two tails then he wins 20 \$ and he continues the game. He flips the coin three times:
     - If he obtains three heads or three tails then we wins 30 \$ and the game is over.
     - Otherwise the game is over.
   - Otherwise the game is over.

   (a) Prove that the game is not a fair one.

   (b) By what amount we must replace the 20 \$ to have a fair game?

3. The joint probability distribution of two random variables X and Y is given by:

$$f(x,y) = \begin{cases} \dfrac{ay}{x^3}, & x > 2,\ 0 < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) Evaluate a.

   (b) Find the marginal probability distributions of X and Y. Deduce if X and Y are independent or not.

   (c) Find μ_X, σ_X, μ_Y, σ_Y, σ_{XY} and ρ_{XY}.

   (d) Compute $P(\mu_X - 2\sigma_X < X < \mu_X + 2\sigma_X)$ and compare with the result given by Chebyshev's theorem.

⟶

4. We consider two dice, one blue and one red. We assume that the blue die is fair and that the red die is loaded in such a way that $P(\text{even}) = 4\,P(\text{odd})$. We also consider a box which contains 180 red balls and 120 blue balls. Now we consider the following two (consecutive) statistical experiments:

   - **First experiment:** We toss the two dice 10 times.
   - **Second experiment:** We select n balls from the box, where n is number of times that we obtain a prime sum in the first experiment.

   (a) Find the probability that we obtain exactly 8 times a prime sum in the first experiment.

   (b) Find the probability we obtain exactly 8 times a prime sum in the first experiment and that we obtain exactly 7 red balls in the second experiment.

   (c) Find the probability that we obtain exactly 7 red balls in the second experiment.

   (d) Find the probability that, in the second experiment, we obtain a number of red balls equals to the triple of the number of blue balls.

   (e) *Bonus question.* Now let X be the number of obtained red balls in the second experiment. Find the probability distribution of X.

MARKS : 1. [15] **2.** [25] **3.** [25] **4.** [35] Bonus question [10]

Lebanese American University	Summer I
Byblos	2006

Probability and Statistics **Date:** 08/09/2006
Test #2 **Duration:** 1h 30

Nota Bene. Use 5 digits in the fractional part of your answers.

1. The probability that a student pilot passes the written test for a private pilot's license is 0.8. Find the probability that the students will pass the test before the fifth try.

2. It is estimated that 98% of the residents of a town are favor to an interdiction of smoking in the train station. If 100 persons of the residents of this town are selected at random and asked their opinion, what is the probability that at most 5 are against to the interdiction of smoking in the train station?

3. A coin is biased so that $P(\text{head}) = 0.7$. We consider the following two (consecutive) statistical experiments:

   - **First experiment:** We toss a die 10 times.
   - **Second experiment:** We flip the (biased) coin n times, where n is number of times that we obtain an even number in the first experiment.

   (a) Find the probability that we obtain two times 1, two times 3, three times 4 and three times 6 in the first experiment.

   (b) Find the probability that we obtain exactly 7 even numbers in the first experiment.

   (c) Find the probability we obtain exactly 7 even numbers in the first experiment and that the number of obtained heads in the second experiment is less than 5.

   (d) Find the probability that the number of heads obtained in the second experiment is 9.

   (e) *Bonus question.* We consider Y the number of heads obtained in the second experiment. Find the probability distribution of Y.

4. The joint probability distribution of two random variables X and Y is given by:

$$f(x,y) = \begin{cases} a(x^2+y^2), & 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) Evaluate a.

   (b) Find the marginal probability distributions of X and Y. Deduce if X and Y are independent or not. $\longrightarrow$

(c) Find μ_X and σ_X.

(d) Compute $P(\mu_X - 3\sigma_X < X < \mu_X + 3\sigma_X)$ and compare with the result given by Chebyshev's theorem.

(e) Now we consider the random variable Z defined by $Z = \min\{X, Y\}$. Find the cumulative distribution of Z.

MARKS : 1. [20] **2.** [20] **3.** [30] **4.** [30] Bonus question [10]

Lebanese American University	**Summer II**
Byblos	**2007**

Probability and Statistics	**Date:** 03/09/2007
Test #2	**Duration:** 1h 30

Nota Bene.

- *Use 5 digits in the fractional part of your answers.*
- *In all the probability questions of exercises 1 and 4, you must use a discrete probability distribution by giving its name.*

1. The probability that a person, living in a certain city, owns a car is estimated to be 0.7. Find the probability that the tenth person randomly interviewed in the city is the fifth one to own a car?

2. We consider the following gambling game. First the player must pay 15 $ to play the game. The player begins the game by flipping a coin two times:
   - If he obtains two heads or two tails then he wins 20 $ and he continues the game. He tosses a die two times:
     - If he obtains same two numbers then he wins 30 $ and the game is over.
     - Otherwise the game is over.
   - Otherwise the game is over.

   (a) Prove that the game is not a fair one.

   (b) By what amount we must replace the 20 $ to have a fair game?

3. The joint probability distribution of two random variables X and Y is given by:

$$f(x,y) = \begin{cases} a(x+2y), & 0 < x < 1,\ 0 < y < 2 \\ 0, & \text{elsewhere.} \end{cases}$$

   (a) Evaluate a.

   (b) Find the marginal probability distributions of X and Y. Deduce if X and Y are independent or not.

   (c) Find μ_X, σ_X, μ_Y, σ_Y, σ_{XY} and ρ_{XY}.

   (d) Compute $P(\mu_X - 2\sigma_X < X < \mu_X + 2\sigma_X)$ and compare with the result given by Chebyshev's theorem.

⟶

$\sigma_{xy} = E(xy) - E(x)E(y)$

$E(xy) = \int_0^1 \int_0^1 xy$ —

4. We consider two dice, one blue and one red. We assume that the red die is loaded in such away that $P(\text{odd}) = 4\,P(\text{even})$ and that the blue die is loaded in such a way that $P(\text{even}) = 3\,P(\text{odd})$. We also consider a loaded coin such that $P(\text{head}) = 2\,P(\text{tail})$. Now we consider the following two (consecutive) statistical experiments:

- **First experiment:** We toss the two dice 8 times.
- **Second experiment:** We flip the coin several times to obtain $k+1$ times head for the first time, where k is the number of times that we obtain an even product in the first experiment.

(a) Find the probability that we obtain exactly 3 times an even product in the first experiment.

(b) Find the probability we obtain exactly 3 times an even product in the first experiment and that we flipped the coin 5 times in the second experiment.

(c) Find the probability that we flipped the coin 5 times in the second experiment.

(d) Find the probability that, in the second experiment, we obtain a number of heads equals to three times the number of tails.

(e) *Bonus question.* Now let X be the number of times that we flipped the coin in the second experiment. Find the probability distribution of X.

MARKS : 1. [15] **2.** [25] **3.** [25] **4.** [35] Bonus question [10]

Lebanese American University **Spring 2008**

Byblos

Probability and Statistics **Date:** 26/05/2008
Test #2 **Duration:** 1h 30

Nota Bene.

- *Use 4 digits in the fractional part of your answers.*
- *In all the probability questions of exercises 1 and 2, you must use a discrete probability distribution by giving its name.*

$\frac{1}{3}$ $\frac{21}{3} = 7$ per 3 weeks

1. A computer crashes, on average, once every 3 days. What is the probability of there being at most 4 crashes in 3 weeks?

   $P(Y \leq 4) = P(4; 7)$

2. In Lebanon, the probability that one or more car accidents will occur during any given month is 0.6.

   (a) Find the probability that in 2008, there will be at most four months in which at least one accident occurs.

   (b) Find the probability that in 2008, April will be the second month in which at least one accident occurs.

   (c) Find the probability that there will be at most three months in which no accidents occur before the third month in which at least one accident occurs.

3. We consider the following gambling game. First the player must pay 20 $ to play the game. The game consists on two steps:

   - **Step 1:** The player flips a coin four times:
     - If he obtains four heads or four tails then he wins 40 $ and he goes to Step 2.
     - If he obtains two head and two tails then he wins 25 $ and he goes to Step 2.
     - Otherwise the game is over (that is, no Step 2).
   - **Step 2:** The player tosses a die three times:
     - If he obtains same three numbers then he wins 45 $ and the game is over.
     - Otherwise the game is over.

   (a) Prove that the game is not a fair one.

   (b) By what amount we must replace the 20 $ that the player must pay at the beginning, to have a fair game?

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4. The joint probability distribution of two random variables X and Y is given by:

$$f(x,y) = \begin{cases} a(2x+y), & 0<x<1,\ 0<y<1 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Evaluate a.

(b) Find the marginal probability distributions of X and Y. Deduce if X and Y are independent or not.

(c) Find the conditional density $f(x|y)$ and deduce $P(X > 0.2|Y = 0.5)$.

(d) Find μ_X and σ_X.

(e) Compute $P(\mu_X - 2\sigma_X < X < \mu_X + 2\sigma_X)$ and compare with the result given by Chebyshev's theorem.

5. *Bonus question.* Let X be a random variable and let $Y = a + bX$ where a and b are two real numbers. We denote by σ_X the standard deviation of X.

(a) Find the covariance σ_{XY} in term of b and σ_X.

(b) Deduce the value of the correlation coefficient ρ_{XY}.

MARKS : 1. [10] **2.** [30] **3.** [25] **4.** [35] Bonus question [10]